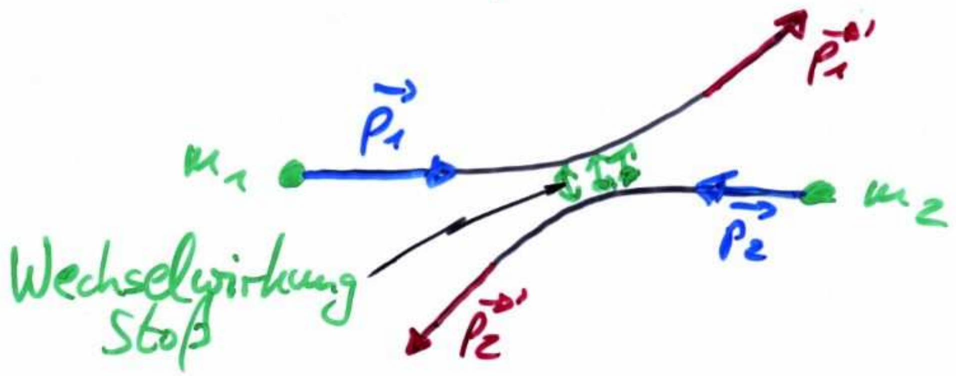
6.1 Stöße (zwischen zwei Teilchen)



$$\boxed{\vec{p}_1{}' + \vec{p}_2{}' = \vec{p}_1 + \vec{p}_2}$$ Impulssatz (ohne äußere Kräfte)

$$E_{kin}' = \frac{\vec{p}_1{}'^2}{2m_1} + \frac{\vec{p}_2{}'^2}{2m_2} = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} + Q$$ Energiesatz

$$E_{kin} = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} = \frac{\vec{p}_{1S}^{\,2}}{2\mu} + \frac{\vec{P}^2}{2M} \quad ; \quad \vec{p}_{1S} = -\vec{p}_{2S}$$

$$\longrightarrow \quad \frac{\vec{p}_{1S}{}'^2}{2\mu} + \frac{\vec{P}^2}{2M} = \frac{\vec{p}_{1S}^{\,2}}{2\mu} + \frac{\vec{P}^2}{2M} + Q$$

$$\longrightarrow \quad \boxed{\frac{\vec{p}_{1S}{}'^2}{2\mu} = \frac{\vec{p}_{1S}^{\,2}}{2\mu} + Q}$$ Energiesatz

Arten von Stößen

- $Q = 0$ elastische Stöße
- $Q < 0$ inelastische Stöße (kinet. Energie $\rightarrow$ innere Energie)
- $Q > 0$ superelastische Stöße (innere Energie $\rightarrow$ kinet. Energie)

NB: inelastische/suprelastische Stöße
$\Leftrightarrow$ ein Stoßpartner hat innere Struktur

- Zentrale Stöße 
- Nicht-zentrale Stöße 

Zentrale Stöße

- elastischer Stoß ($Q=0$)

wähle ruhenden Stoßpartner: $p_2 = 0$

$$p_1 = p_1' + p_2' \Rightarrow m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$\frac{p_1^2}{2m_1} = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} \Rightarrow m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{m_1}{m_2}(v_1 - v_1')$$

$$m_1 v_1^2 = m_1 v_1'^2 + m_2 \left[\frac{m_1}{m_2}(v_1 - v_1')\right]^2$$

$$m_1 (v_1^2 - v_1'^2) = \frac{m_1^2}{m_2}(v_1 - v_1')^2$$

$$m_2 (v_1^2 - v_1'^2) = m_1 (v_1 - v_1')^2 \quad \Big| \cdot \frac{1}{v_1 - v_1'}$$

$$m_2 (v_1 + v_1') = m_1 (v_1 - v_1')$$

$\rightarrow \dots \rightarrow$

$$\boxed{v_1' = \frac{m_1 - m_2}{m_1 + m_2} \cdot v_1 \qquad v_2' = \frac{2 m_1}{m_1 + m_2} \cdot v_1}$$

$v_1' \rightarrow$ in $v_2' = \dots$ eingesetzt $\rightarrow$

$$\rightarrow p_1 = m_1 v_1 = m_1 v_1' + m_2 v_2' = \underbrace{\frac{m_1 - m_2}{m_1 + m_2} m_1 v_1}_{} + \frac{2 m_1 m_2}{m_1 + m_2} v_1 = p_1' + p_2'$$

$$\rightarrow \boxed{p_1 = p_1' + 2\mu \cdot v_1} \rightarrow p_2' = 2\mu v_1$$

Beispiele:

- $m_1 = m_2 =: m \longrightarrow v_1' = 0 \cdot v_1 = 0$

  $v_1 \neq 0,\ v_2 = 0 \qquad v_2' = \frac{2m}{m+m} v_1 = v_1$

- $m_1 = \frac{1}{2} m_2 \rightarrow v_1' = \frac{\frac{1}{2} m_2 - m_2}{\frac{1}{2} m_2 + m_2} \cdot v_1 = \frac{-\frac{1}{2} m_2}{\frac{3}{2} m_2} v_1 = -\frac{1}{3} v_1$

  $v_2' = \frac{m_2}{\frac{1}{2} m_2 + m_2} \cdot v_1 = \frac{1 m_2}{\frac{3}{2} m_2} v_1 = \frac{2}{3} v_1$

- $m_1 = 2 m_2 \longrightarrow v_1' = \frac{2m_2 - m_2}{2m_2 + m_2} v_1 = \frac{1 m_2}{3 \cdot m_2} v_1 = \frac{1}{3} v_1$

  $v_2' = \frac{4 m_2}{3 m_2} v_1 = \frac{4}{3} v_1$

- Energieübertrag $1 \rightarrow 2$:

$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$$\Delta E_{kin} = \frac{p_2'^2}{2m_2} = \frac{2\mu^2}{m_2} v_1^2 = \frac{(m_1 m_2)^2}{(m_1+m_2)^2} \cdot \frac{2 v_1^2}{m_2}$$

$$= \frac{4 m_2 m_1}{(m_1+m_2)^2} \cdot \frac{m_1 v_1^2}{2} = \frac{4 m_1 m_2}{(m_1+m_2)^2} \cdot E_{kin,1}$$

$$\longrightarrow \boxed{\Delta E_{kin} = \frac{4 m_1 m_2}{(m_1+m_2)^2} \cdot E_{kin,1} = \frac{4\mu}{(m_1+m_2)} \cdot E_{kin,1}}$$

- maximal inelastische / total unelastische zentrale Stöße

d.h. $v_1' = v_2' =: v'$

$p_2 = m_2 v_2 = 0$: $\quad m_1 v_1 = m_1 v_1' + m_2 v_2' = (m_1 + m_2) v'$

$$Q + \frac{m_1 v_1^2}{2} = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2} = \frac{1}{2}(m_1 + m_2) v'^2$$

$$\longrightarrow \quad \boxed{v' = \frac{m_1}{m_1 + m_2} v_1} \quad \text{und} \quad \boxed{Q \overset{\text{s.u.}}{=} \ldots = -\underbrace{\frac{\mu}{2} v_1^2}}$$

Zunahme der inneren Energie

innere kinet. Energie bzgl. Schwerpunktes

$$Q = \frac{m_1 + m_2}{2} v'^2 - \frac{m_1 v_1^2}{2} = \left(\frac{m_1 + m_2}{2} \cdot \frac{m_1^2}{(m_1 + m_2)^2} - \frac{m_1}{2} \right) v_1^2$$

$$= \frac{m_1}{2} \left(\frac{m_1}{m_1 + m_2} - 1 \right) v_1^2 = -\frac{m_1 m_2}{m_1 + m_2} \cdot \frac{v_1^2}{2} = -\frac{\mu}{2} v_1^2$$