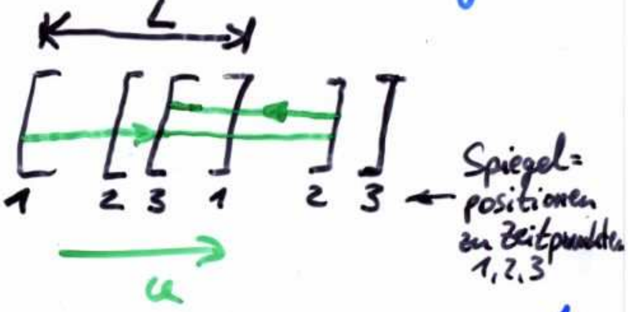
- **Längenkontraktion**

- mitbewegtes System S'
→ ruhende Uhr

$\Delta t' = \frac{2L'}{c}$

- ruhendes System S
→ mit $u \neq 0$ bewegte Uhr



$ct_{12} = L + ut_{12} \to t_{12} = \frac{L}{c-u}$

$ct_{23} = L - ut_{23} \to t_{23} = \frac{L}{c+u}$

$\to \Delta t = t_{12} + t_{23} = L\left(\frac{1}{c-u} + \frac{1}{c+u}\right)$

$= L \cdot \frac{(c+u)+(c-u)}{(c-u)(c+u)} = L\frac{2c}{c^2-u^2}$

$\to \Delta t = \frac{2L}{c} \frac{1}{1-\frac{u^2}{c^2}} = \frac{2L}{c}\gamma^2$

Zeitdilatation $\Delta t = \Delta t' \cdot \gamma \to \Delta t' \cdot \gamma = \frac{2L'}{c} \cdot \gamma = \frac{2L}{c}\gamma^2$

$\to L' = L \cdot \gamma$

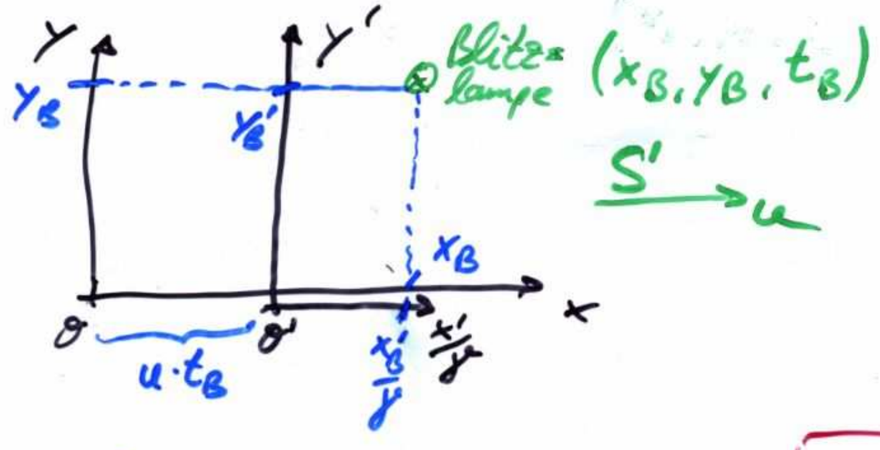
$\to \boxed{L = L'/\gamma = L'\sqrt{1-\frac{u^2}{c^2}}}$

Längenkontraktion ($\gamma \geq 1$)

→ Im ruhenden Inertialsystem erscheint ein bewegter Maßstab verkürzt.

7.2 Lorentztransformation

- Ortstransformation



$$\rightarrow \frac{x_B'}{\gamma} = x_B - u \cdot t_B \longrightarrow \boxed{x_B' = \gamma\left[x_B - \frac{u}{c} \cdot ct_B\right]}$$

$$y_B' = y_B \longrightarrow \boxed{y_B' = y_B \;,\; z_B' = z_B}$$

NB: Umkehrtransformation $x_B = \gamma\left[x_B' - \frac{(-u)}{c} \cdot ct_B'\right]$

- Zeittransformation

aus Umkehrtransf. $x_B = \gamma\left[x_B' + \frac{u}{c} \cdot ct_B'\right]$

$$\rightarrow ct_B' = \frac{c}{u}\left[\frac{x_B}{\gamma} - x_B'\right] = \frac{c}{u}\left[\frac{x_B}{\gamma} - \gamma\left[x_B - \frac{u}{c} \cdot ct_B\right]\right]$$

$$= \gamma \frac{c}{u}\left[\frac{x_B}{\gamma^2} - x_B + \frac{u}{c} \cdot ct_B\right] = \gamma \frac{c}{u}\left[x_B \underbrace{\left(\frac{1}{\gamma^2} - 1\right)}_{\left(1-\frac{u^2}{c^2}\right)-1 = -\frac{u^2}{c^2}} + \frac{u}{c} \cdot ct_B\right]$$

$$= \gamma \frac{c}{u}\left[-\frac{u^2}{c^2} \cdot x_B + \frac{u}{c} \cdot ct_B\right]$$

$$\longrightarrow \boxed{ct_B' = \gamma\left[ct_B - \frac{u}{c} x_B\right]}$$

Matrixdarstellung

$$\begin{pmatrix} ct_B' \\ x_B' \\ y_B' \\ z_B' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{4\times 4\text{-Matrix}} \cdot \underbrace{\begin{pmatrix} ct_B \\ x_B \\ y_B \\ z_B \end{pmatrix}}_{\text{Vierer-Vektor}} \quad \text{mit } \beta := \frac{u}{c}$$

kurz: $$\begin{pmatrix} ct_B' \\ x_B' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct_B \\ x_B \end{pmatrix}$$

- ## Relativistische Geschwindigkeitsaddition

$$v_x' = \frac{dx'}{dt'} = \frac{dx'}{dt} \cdot \frac{dt}{dt'} = \frac{d}{dt}\left(\gamma\left[x - \frac{u}{c}\cdot ct\right]\right) \cdot \frac{dt}{dt'}$$

$$= \gamma\left[\frac{dx}{dt} - \frac{u}{c}\cdot c\frac{dt}{dt}\right] \cdot \frac{dt}{dt'}$$

$$= \gamma\left[v_x - \frac{u}{c}\cdot c\right] \cdot \frac{dt}{dt'} = \gamma\left[v_x - u\right] \cdot \frac{dt}{dt'}$$

dabei ist:

$$\frac{dt'}{dt} = \frac{d}{dt}\left(\frac{ct'}{c}\right) = \frac{d}{dt}\left(\frac{1}{c}\gamma\left[ct - \frac{u}{c}x\right]\right) = \ldots = \gamma\left[1 - \frac{u}{c^2}v_x\right]$$

$$\longrightarrow v_x' = \gamma \cdot \left[v_x - u\right] \frac{1}{\gamma\left[1 - \frac{u}{c^2}v_x\right]} \longrightarrow \boxed{v_x' = \frac{v_x - u}{1 - \frac{u}{c^2}v_x}}$$

$$v_y' = \frac{dy'}{dt'} = \frac{dy}{dt}\cdot\frac{dt}{dt'} = v_y \frac{1}{\gamma\left[1-\frac{u}{c^2}v_x\right]}$$

$$\rightarrow \boxed{v_y' = v_y \cdot \frac{1}{\gamma\left[1-\frac{u}{c^2}v_x\right]} \; ; \; v_z' = v_z \cdot \frac{1}{\gamma\left[1-\frac{u}{c^2}v_x\right]}}$$

NB: ▸ Für $u \ll c$: $v_x' = \frac{v_x - u}{1-\frac{u v_x}{c^2}} \rightarrow v_x' \approx v_x - u$

$v_y' = \dots \rightarrow v_y' \approx v_y$

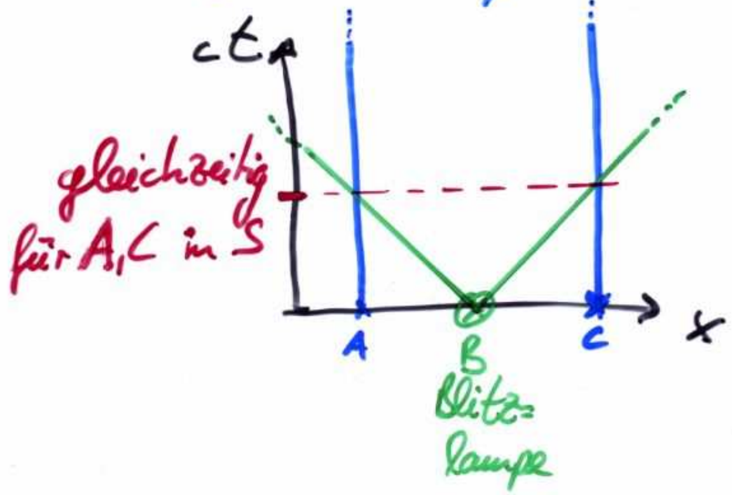
$v_z' = \dots \rightarrow v_z' \approx v_z$

$\Big\} \hat{=}$ Galilei-Transformation

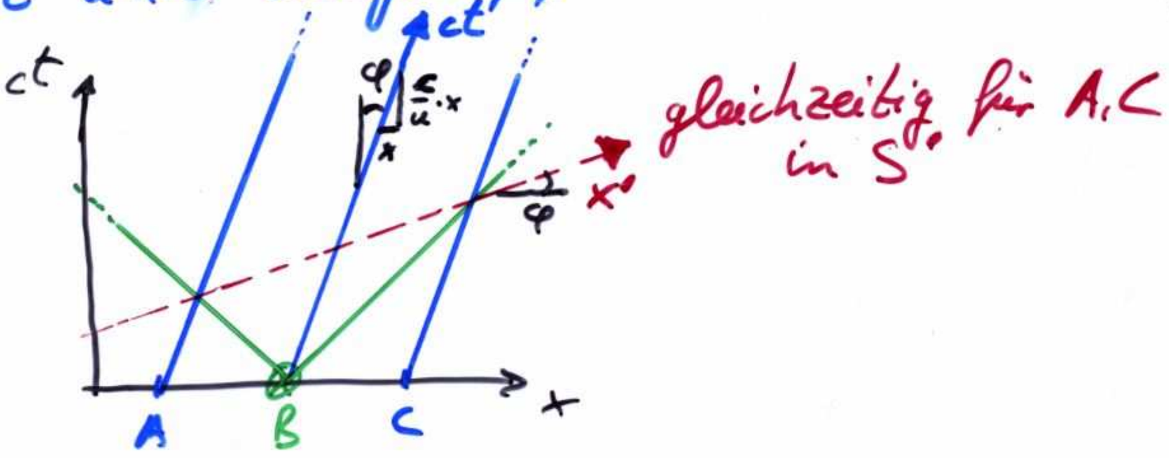
▸ $v_x = c \rightarrow v_x' = \frac{c-u}{1-\frac{u}{c^2}\cdot c} = \frac{c\left(1-\frac{u}{c}\right)}{1-\frac{u}{c}} = c$!

7.3 Gleichzeitigkeit

• ruhendes System S



• mit $u \neq 0$ bewegtes System S'



Für mitbewegte Beobachter A, C
erreicht Lichtblitz A und C gleichzeitig!

→ $ct' = \text{const}$ entlang gestrichelter Linie (x'-Achse)

- x', ct'-Achsen sind gegenüber x, ct-Achse geneigt
- x', ct'-Achsen sind nicht orthogonal zueinander
- Winkel zwischen x, x'- und ct, ct'-Achse: $\tan \varphi = \frac{u}{c}$