γv hat Eigenschaft einer Lorentz-Transformation $\longrightarrow$ relativist. Impuls:

$$\boxed{p' := m\gamma' v' = \gamma_u \left[m\gamma v - \beta_u m\gamma c \right] = \gamma_u \left[p - \beta_u \cdot p_0 \right]}$$

Lorentztransformation des relativist. Impulses

mit $p_0 := m\gamma c$ als Komponente des

Impuls-Vierervektors
$$\begin{pmatrix} p_0 \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma m c \\ \gamma m v_x \\ \gamma m v_y \\ \gamma m v_z \end{pmatrix}$$

$$\left(\gamma := \frac{1}{\sqrt{1 - \frac{v_x^2 + v_y^2 + v_z^2}{c^2}}} \right)$$

$$\boxed{p_0' = m\gamma' c = \gamma_u \left[p_0 - \beta_u \cdot p \right]}$$

Lorentztransf. der relativist. Energie $E_{rel} = p_0 c$

$$\Big\} \longrightarrow \boxed{\begin{pmatrix} p_0' \\ p' \end{pmatrix} = \begin{pmatrix} \gamma_u & -\beta_u \gamma_u \\ -\beta_u \gamma_u & \gamma_u \end{pmatrix} \begin{pmatrix} p_0 \\ p \end{pmatrix}}$$

mit relativistischem Impuls $\vec{p} := \gamma m \vec{v}$

und relativistischer Energie E_{rel} $p_0 := \gamma m c = \frac{E_{rel}}{c}$

- ## Impulserhaltung (1. Newtonsche Axiom)

▶ klassisch: $\dot{p} = \frac{dp}{dt} = 0 \xrightarrow[v \to v' = v - u]{S \to S'} \dot{p}' = \underbrace{\dot{p}}_{=0} - m\underbrace{\dot{u}}_{=0} = 0$

$\hookrightarrow p \to p' = p - mu$

▶ relativistisch: $\dot{p} = 0 \xrightarrow{S \to S'} \dot{p}' = \frac{dp'}{dt}$

$$\frac{dp'}{dt} = \frac{d}{dt}\left(\gamma_u\left[p - \beta_u p_0\right]\right) \underset{\uparrow}{=} \gamma_u\left[\underbrace{\dot{p}}_{\substack{=0 \\ (\text{s.o.})}} - \beta_u \underbrace{\dot{p}_0}_{=0}\right]$$

$u = \text{const}$

$\hookrightarrow \beta_u = \frac{u}{c} = \text{const}$

$\hookrightarrow \gamma_u = \text{const}$

$\dot{p}_0 = \frac{d}{dt}(\gamma mc) = 0$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$v = \text{const} \to \dot{\gamma} = 0$

- ## 2. Newtonsches Axiom

▶ klassisch: $F = \frac{dp}{dt} = \frac{d(mv)}{dt} \overset{m=\text{const}}{=} m\frac{dv}{dt} = ma$

▶ relativistisch: $F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m v)$

$$\overset{m=\text{const}}{=} \dot{\gamma} m v + \gamma m \dot{v}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \to \frac{d\gamma}{dt} = \frac{d\gamma}{dv} \cdot \frac{dv}{dt} = \frac{v}{c^2}\gamma^3 \cdot \dot{v} \Big\} \to = \frac{v^2}{c^2} \cdot m\gamma^3 \cdot \dot{v} + \gamma m \dot{v}$$

$$= \gamma^3 m \dot{v} \cdot \Big(\frac{v^2}{c^2} + \underbrace{\frac{1}{\gamma^2}}_{=1-\frac{v^2}{c^2}}\Big)$$

$$\hookrightarrow \boxed{F = \gamma^3 m \dot{v} = \gamma^3 m a}$$

[Minkowski-Kraft $K := \gamma F$]

NB: allgemeiner Ausdruck für relativist. Kraft $\vec{F}$ ist komplizierter

- Energieerhaltung, kinetische Energie

▸ klassisch: $E_{kin} = W = \int F dx = \int m a \cdot \frac{dx}{dt} dt = \int m \frac{dv}{dt} \cdot v \, dt$

$$= \int m v \, dv = \frac{1}{2} m v^2$$

▸ relativistisch:

$$W_x = \int F_x dx = \int \frac{dp_x}{dt} dx = \int m\gamma^3 \frac{dv}{dt} \cdot \underbrace{\frac{dx}{dt}}_{=v} dt$$

$$= \int m\gamma^3 \frac{dv}{dt} \cdot v \, dt = mc^2 \int_0^{v_x/c} \gamma^3 \frac{v}{c} d\left(\frac{v}{c}\right) = \dots =$$

$$= mc^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \right]_0^{v_x}$$

$$\boxed{W = E_{kin} = mc^2(\gamma - 1)} = \underbrace{\gamma mc^2}_{\text{relativistische Energie } E_{rel}} - \underbrace{mc^2}_{\text{Ruheenergie}}$$

$$\boxed{E_{rel} := \gamma mc^2} = p_0 \cdot c$$

▸ Übergang zu nicht-relativist. Fall: $v \ll c$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \dots$$

(Taylor-Reihenentwicklung)

$$\boxed{E_{kin} = mc^2 \Big(\underbrace{1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \dots}_{=\gamma} - 1 \Big) = \frac{1}{2} m v^2 + \dots}$$

- relativistischer Energiesatz

Betrachte: $\left(\frac{E}{c}\right)^2 - \left(\vec{p}\right)^2 = \left(\frac{E}{c}\right)^2 - p_x^2 - p_y^2 - p_z^2$

(NB: $E \equiv E_{rel}$)

$$= (\gamma mc)^2 - (\gamma m v_x)^2 - (\gamma m v_y)^2 - (\gamma m v_z)^2$$

$$= \gamma^2 m^2 c^2 \left[1 - \left(\frac{v_x}{c}\right)^2 - \left(\frac{v_y}{c}\right)^2 - \left(\frac{v_z}{c}\right)^2\right]$$

$$= \gamma^2 m^2 c^2 \underbrace{\left[1 - \frac{\vec{v}^2}{c^2}\right]}_{= 1/\gamma^2}$$

$\rightarrow$ $\boxed{\left(\frac{E}{c}\right)^2 - \left(\vec{p}\right)^2 = m^2c^2}$ = const in allen Inertialsystemen, d.h. <u>Lorentz-invariant</u>

$\rightarrow$ <u>Masse m</u> (bzw. m^2c^2) ist <u>Lorentz-invariante Größe</u>

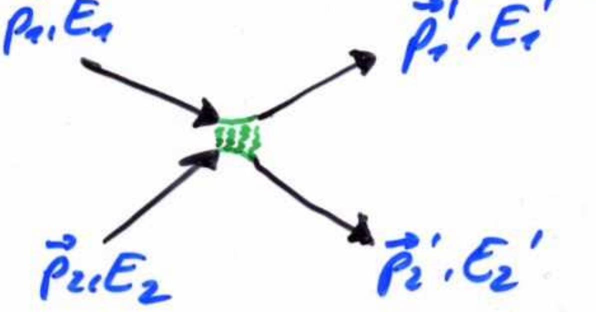
$\rightarrow$ nützliche Relation ($E = E_{rel}$)

- $\gamma = \frac{E}{mc^2}$
- $\beta = \frac{pc}{E}$
- $\gamma\beta = \frac{p}{mc}$

7.7 Anwendung relativist. Dynamik

z.B. Stöße zwischen Teilchen mit relativist. Energien

- Wie bei nicht-relativist. Stößen gilt:



▶ **Impulssatz:** $\vec{p}_1 + \vec{p}_2 = \vec{p}_1{}' + \vec{p}_2{}'$

$$m_1 \gamma_1 \vec{v}_1 + m_2 \gamma_2 \vec{v}_2 = m_1 \gamma_1' \vec{v}_1{}' + m_2 \gamma_2' \vec{v}_2{}'$$

▶ **Energiesatz:** (elastische Stöße) $E_1 + E_2 = E_1' + E_2'$

$$\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma_1' m_1 c^2 + \gamma_2' m_2 c^2$$

sind beides wesentliche Arbeitswerkzeuge zur Behandlung relativistischer Stöße!