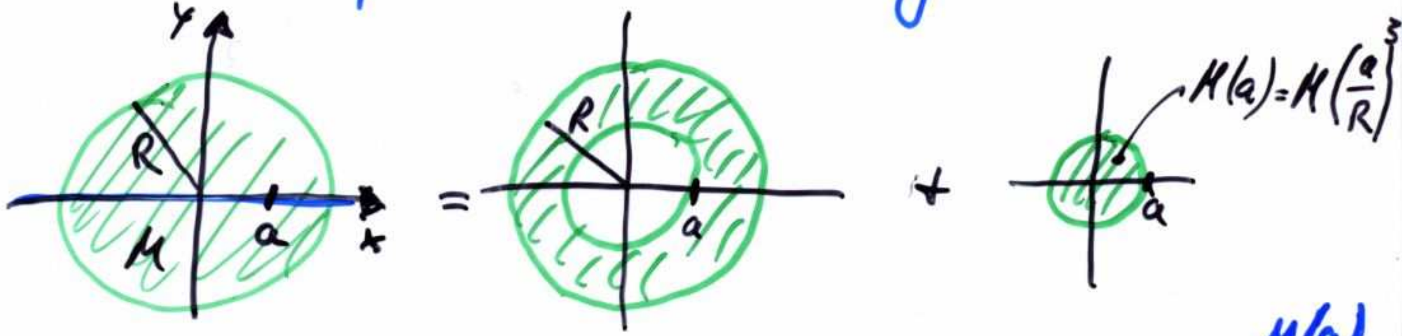
- Gravitationspotential in homogener Kugel



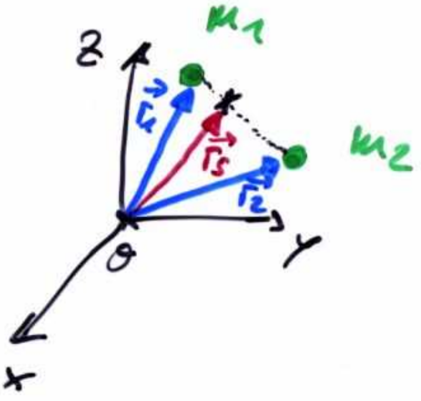
$$E_{pot} = \text{const} \qquad E_{pot} = -G_N \frac{M(a)}{a} = -G_N \frac{M a^2}{R^3}$$

$$\longrightarrow E_{pot,ges}(a) = \text{const} + \left(-G_N \frac{M a^2}{R^3}\right)$$

$$\longrightarrow \vec{F} = -\text{grad}\, E_{pot,ges}(a) = \ldots = \begin{pmatrix} +G_N \frac{2Ma}{R^3} \\ \vdots \end{pmatrix}$$

6 Systeme von Massenpunkten

- **Schwerpunkt**



$$\boxed{\vec{r}_S := \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{1}{M}\sum_{i=1}^{N} m_i \cdot \vec{r}_i}$$

$$M := \sum_{i=1}^{N} m_i$$

- **Schwerpunktsgeschwindigkeit**

$$\boxed{\vec{v}_S = \dot{\vec{r}}_S = \frac{d\vec{r}_S}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \dot{\vec{r}}_i = \frac{1}{M}\sum_{i=1}^{N} m_i \cdot \vec{v}_i}$$

- **Schwerpunktimpuls**

$$\boxed{\vec{P} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i = M \cdot \vec{v}_S}$$

- **Impulserhaltungssatz** (ohne äußere Kräfte)

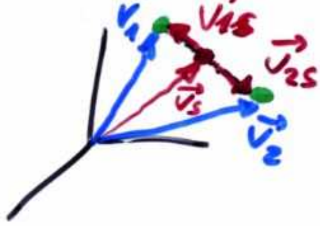
$$\boxed{\vec{P} = \sum_{i=1}^{N} \vec{p}_i = \text{const} \rightarrow \sum_{i=1}^{N} \vec{p}_{iS} := \left(\sum_{i=1}^{N} \vec{p}_i\right) - \vec{P} = 0}$$

- **Schwerpunktsatz** (mit äußeren Kräften)

$$\boxed{\vec{F}_a = \sum_{i=1}^{N} \vec{F}_{ai} = \sum_{i=1}^{N} \frac{d}{dt}\vec{p}_i = \frac{d}{dt}\left(\sum_{i=1}^{N} \vec{p}_i\right) = \frac{d}{dt}\vec{P} = M\frac{d\vec{v}_S}{dt} =: M\vec{a}_S}$$

Bewegung des Schwerpunkts S so als ob alle Masse in S konzentriert und alle Kräfte in S angreifen

- <u>kinetische Energie</u> $\left(\frac{1}{2} m v^2\right)$



betrachte: $\vec v_i = \vec v_{is} + \vec v_s$

$\vec p_i = m_i \vec v_i$, $\vec P = M \cdot \vec v_s \rightarrow \vec p_i = m_i \vec v_i = m_i \cdot \vec v_{is} + m_i \cdot \vec v_s$

$$= m_i \vec v_{is} + m_i \frac{M}{M} \vec v_s$$

$$= m_i \vec v_{is} + \frac{m_i}{M} \vec P$$

$$\rightarrow \quad \vec p_i = \vec p_{is} + \frac{m_i}{M} \vec P$$

$$\rightarrow \sum_{i=1}^{N} \vec p_{is} = \sum_{i=1}^{N} \left(\vec p_i - \frac{m_i}{M} \vec P \right) = \sum_{i=1}^{N} \vec p_i - \sum_{i=1}^{N} \frac{m_i}{M} \vec P$$

$$= \sum_{i=1}^{N} \vec p_i - \frac{\vec P}{M} \underbrace{\sum m_i}_{=M}$$

$$= \left(\sum_{i=1}^{N} \vec p_i \right) - \vec P = 0$$

$$\rightarrow E_{kin} = \sum_{i=1}^{N} E_{kin,i} = \sum_{i=1}^{N} \frac{m_i}{2} \vec v_i^{\,2} = \sum_{i=1}^{N} \frac{m_i^2 \vec v_i^{\,2}}{2 m_i} = \sum_{i=1}^{N} \frac{\vec p_i^{\,2}}{2 m_i}$$

$$= \sum_{i=1}^{N} \frac{1}{2 m_i} \left[\vec p_{is} + \frac{m_i}{M} \vec P \right]^2$$

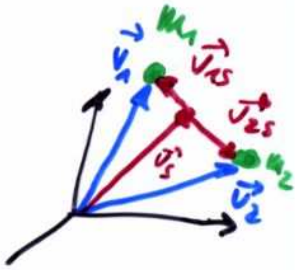
$$= \sum_{i=1}^{N} \frac{1}{2 m_i} \left[\vec p_{is}^{\,2} + \frac{m_i^2}{M^2} \vec P^2 + 2 \frac{m_i}{M} \vec P \cdot \vec p_{is} \right]$$

$$= \underbrace{\sum_{i=1}^{N} \frac{\vec p_{is}^{\,2}}{2 m_i}} + \frac{\vec P^2}{M^2} \underbrace{\sum_{i=1}^{N} \frac{m_i^{\cancel{2}}}{2 \cancel{m_i}}}_{= \frac{M}{2}} + \frac{\vec P}{M} \underbrace{\sum_{i=1}^{N} \frac{\cancel{m_i} \vec p_{is}}{\cancel{m_i}}}_{= \sum \vec p_{is} = 0}$$

$$\rightarrow \boxed{E_{kin} = \underbrace{E_{kin}^{(s)}}_{\text{bezüglich Schwerpunkt}} + \underbrace{\frac{\vec P^2}{2M}}_{\text{des Schwerpunkts}}}$$

<u>kinetische Energie</u>

- reduzierte Masse im Zweikörpersystem



$$\sum \vec p_{iS} = 0 = \vec p_{1S} + \vec p_{2S} = 0 \rightarrow \vec p_{1S} = -\vec p_{2S} =: \vec p_{12}$$

$$E_{kin}^{(S)} = \frac{\vec p_{1S}^{\,2}}{2m_1} + \frac{\vec p_{2S}^{\,2}}{2m_2} = \frac{\vec p_{12}^{\,2}}{2m_1} + \frac{\vec p_{12}^{\,2}}{2m_2} = \frac{\vec p_{12}^{\,2}}{2} \cdot \left(\frac{1}{m_1} + \frac{1}{m_2} \right)$$

$$= \frac{1}{2} \underbrace{\left(\frac{m_1 + m_2}{m_1 m_2} \right)}_{=: \frac{1}{\mu}} \cdot \vec p_{12}^{\,2}$$

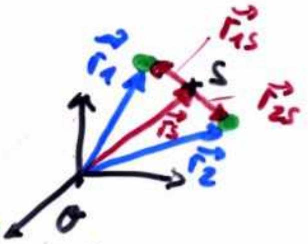
$$\rightarrow \boxed{\mu := \frac{m_1 m_2}{m_1 + m_2}}$$ reduzierte Masse

$$\rightarrow E_{kin}^{(S)} = \frac{1}{2} \frac{p_{12}^2}{\mu} = \frac{1}{2} \mu v_{12}^2 \quad \text{mit } \vec v_{12} := \vec v_1 - \vec v_2$$

$$\Big(\vec v_{12} := \frac{1}{\mu} \vec p_{12} = \frac{m_1 + m_2}{m_1 m_2} \vec p_{1S} = \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \vec p_{1S} = \frac{\vec p_{1S}}{m_1} + \frac{\vec p_{1S}}{m_2}$$

$$= \frac{\vec p_{1S}}{m_1} - \frac{\vec p_{2S}}{m_2} = \vec v_{1S} - \vec v_{2S} = (\vec v_{1S} + \vec v_S) - (\vec v_{2S} + \vec v_S) = \vec v_1 - \vec v_2 \Big)$$

- Drehimpuls



$$\vec L_O = \sum_{i=1}^{N} \vec r_i \times \vec p_i = \sum_{i=1}^{N} m_i (\vec r_i \times \vec v_i) = \sum_{i=1}^{N} m_i (\vec r_{iS} + \vec r_S) \times (\vec v_{iS} + \vec v_S)$$

$= \ldots$ (ausmultiplizieren und vereinfachen) ...

$$\longrightarrow \boxed{\underbrace{\vec L_O}_{\text{bezüglich Ursprung } O} = \underbrace{\vec r_S \times \vec P}_{\text{Schwerpunkt}} + \sum_{i=1}^{N} \vec r_{iS} \times \vec p_{iS} =: \vec r_S \times \vec P + \underbrace{\vec L_S}_{\text{bezüglich Schwerpunkt}}}$$

- ## Drehmoment

$$\vec{D}_O = \frac{d\vec{L}_O}{dt} = \underbrace{\underbrace{\dot{\vec{r}}_S}_{\vec{v}_S} \times \underbrace{\vec{P}}_{\vec{v}_S M}}_{=0} + \vec{r}_S \times \dot{\vec{P}} + \sum \underbrace{\underbrace{\dot{\vec{r}}_{is}}_{\vec{v}_{is}} \times \underbrace{\vec{p}_{is}}_{\vec{v}_{is} m_i}}_{=0} + \sum \vec{r}_{is} \times \dot{\vec{p}}_{is}$$

$$\boxed{\begin{aligned} \vec{D}_O &= \vec{r}_S \times \dot{\vec{P}} + \sum_{i=1}^{N} \vec{r}_{is} \times \dot{\vec{p}}_{is} \\ &= \vec{r}_S \times \vec{F}_a + \sum_{i=1}^{N} \vec{r}_{is} \times \vec{F}_{is} \\ &= \vec{r}_S \times \vec{F}_a + \sum_{i=1}^{N} \vec{r}_{is} \times \vec{F}_{ai} \\ \underbrace{\vec{D}_O}_{\text{bezüglich Ursprung } O} &=: \underbrace{\vec{r}_S \times \vec{F}_a}_{\text{Schwerpunkt}} + \underbrace{\vec{D}_S}_{\text{bezüglich Schwerpunkt } S} \end{aligned}}$$

$\vec{F}_a = \dot{\vec{P}}$ (Schwerpunktsatz)

$\vec{F}_{is} = \vec{F}_{ai} + \sum_{k \neq i} \vec{F}_{ik}$

$\vec{D}_S := \sum_{i=1}^{N} \vec{r}_{is} \times \vec{F}_{ai}$